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Electromagnetic barrier penetration: wave packets and signals

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Abstract. The time delay connected with the transmission of an electromagnetic wave across a barrier is investigated. The stationary phase method implies that the effective group velocity of the transmitted wave, inside the barrier, can exceed the velocity of light *in vacuo*. Consequently the transmission of a signal with a sharp front is studied to determine the upper bound for the signal velocity. The approximation employed does not yield a conclusive answer in general, while for the special case of critical incidence the proof is given that the effective signal velocity does not exceed the velocity of light *in vacuo*.

1. Introduction

The interest in the possibility of faster-than-light propagation has recently been revived owing to speculations about tachyons. De Beauregard and Ricard (1970), de Beauregard *et al* (1971) and de Beauregard (1973) conjectured the tachyonic properties of the evanescent electromagnetic wave.

In dispersive media with refractive index less than unity the phase velocity of an electromagnetic wave exceeds c, the velocity of light *in vacuo*. Even the group velocity can exceed c if the frequencies of the waves in a group are centred in the vicinity of one of the eigenfrequencies of the medium, ie in the region of anomalous dispersion. However, a detailed analysis has shown that the velocity of a signal imposed on the wave cannot exceed c (Brillouin 1960).

These peculiarities of wave propagation are caused by the internal properties of the medium. Analogous phenomena can nevertheless arise, even *in vacuo*. For example, in empty waveguides phase velocities exceed c owing to boundary conditions. An even more striking example is offered by the electromagnetic barrier, ie an optically less dense layer in a medium, if the angle of incidence is greater than the critical angle of total reflection.

The evanescent wave in an electromagnetic barrier was investigated in connection with the Goos–Hänchen shift of the transmitted and reflected beam (Strnad and Kodre 1974). So far, to our knowledge, the signal transmission time for an electromagnetic barrier has not been analysed in detail†.

In the first part of the paper we demonstrate that the effective group velocity for barrier transmission can exceed c. Consequently an investigation of signal velocity is

† The delay time at total reflection on a semi-infinite medium was studied by Agudin (1968). In an earlier paper we investigated the delay time of the beam reflected and transmitted by a barrier using a minimum-marked-wave method and verified the results in the scope of scattering theory. These results agree with the results in the first part of the present paper.

given. Conclusive results can be obtained for the vicinity of the critical angle, which is the most interesting region from the experimental as well as from the theoretical point of view.

2. Wave packets

The boundary planes of a thin layer of a medium with index of refraction n_1 are placed at $z = 0$ and $z = Z$. The layer is surrounded by a medium with index of refraction $n > n_1$. A beam of electromagnetic waves impinges on the boundary $z = 0$ in the plane of incidence xz with the angle of incidence θ. The wave vector of the incident and transmitted wave is then $(n\omega/c)(\sin\theta, 0, \cos\theta)$. Two basic polarization states will be considered: the transverse electric (TE) and the transverse magnetic (TM) polarization. The transverse field in the incident wave packet can be constructed as

$$A = \iint F(\omega, \omega', \theta, \theta') \exp(\mathrm{i}\Phi(\omega', \theta'))\, \mathrm{d}\omega'\, \mathrm{d}\theta'. \tag{2.1}$$

For our purposes the spectral density $F(\omega, \omega', \theta, \theta')$ should have nonzero values in a small region around ω and θ. The phase can be written in the form

$$\Phi(\omega, \theta) = (n\omega/c)(x \sin\theta - z\cos\theta) - \omega t. \tag{2.2}$$

In the approximation of the stationary phase method the group velocity of the packet is obtained as the velocity of propagation of the maximum of the wave packet. The extremal value of the integral (2.1) is, in the scope of this approximation, obtained at the stationary point of the rapidly oscillating term. This point is determined by the conditions

$$\partial\Phi(\omega, \theta)/\partial\omega = 0 \qquad \partial\Phi(\omega, \theta)/\partial\theta = 0. \tag{2.3}$$

For the incident wave packet these conditions give†

$$x = ct\sin\theta/n \qquad z = ct\cos\theta/n,$$

ie the maximum of the incident wave packet reaches the origin $x = 0$, $z = 0$ at the time $t = 0$. Owing to the small width of the spectral density mean values ω and θ have been inserted for the variables ω' and θ'.

In the transmitted wave packet each constituent plane wave has to be ascribed its proper value of the transmission coefficient,

$$T(\omega, \theta) = \sin\delta \exp(-\mathrm{i}nZ\omega\cos\theta/c)/\sin(\delta - \mathrm{i}\alpha\omega). \tag{2.4}$$

Here $-\delta = -2\tan^{-1}(nn_1\epsilon/a^2\cos\theta)$ is the phase shift at total reflection on a semi-infinite medium and $\alpha = n_1\epsilon Z/c$ with $\epsilon = (n^2\sin^2\theta/n_1^2 - 1)^{1/2}$. The introduction of parameter a unifies the expressions for both polarization states: $a = n$ for TE and $a = n_1$ for TM. Thus the transmitted wave packet can be written in the form

$$A_{\mathrm{T}} = \iint F(\omega, \omega', \theta, \theta')|T(\omega', \theta')| \exp(\mathrm{i}\Phi_{\mathrm{T}}(\omega', \theta'))\, \mathrm{d}\omega'\, \mathrm{d}\theta' \tag{2.5}$$

† Thereby the frequency dependence of n is neglected. This is done also in the derivations to follow since the dispersion of n and n_1 is not of interest here and does not introduce any essential effect.

with the phase

$$\Phi_T(\omega, \theta) = (n\omega/c)[x \sin\theta - (z-Z)\cos\theta] - \omega t + \tan^{-1}(\tanh \alpha\omega/\tan\delta). \quad (2.6)$$

Conditions (2.3) imply in this case that

$$x = X + c(t-\tau)\sin\theta/n \qquad z = Z + c(t-\tau)\cos\theta/n,$$

where

$$\tau = \frac{\sin\delta[(n^2-n_1^2)\sin^2\theta \sinh\alpha\omega \cosh\alpha\omega - n_1^2\alpha\omega\cos\delta\cos^2\theta]}{\omega n_1^2\epsilon^2(\sinh^2\alpha\omega + \sin^2\delta)\cos^2\theta} \quad (2.7a)$$

is the time of appearance of the maximum in the transmitted wave packet at the boundary plane $z = Z$ and

$$X = \frac{c\sin\delta\sin\theta[(n^2-n_1^2)\sinh\alpha\omega\cosh\alpha\omega - n^2\alpha\omega\cos\delta\cos^2\theta]}{\omega n n_1^2\epsilon^2(\sinh^2\alpha\omega + \sin^2\delta)\cos^2\theta} \quad (2.7b)$$

is the corresponding longitudinal shift.

A similar procedure, with the transmission coefficient T of (2.4) replaced by the reflection coefficient $R = \sin i\alpha\omega/\sin(i\alpha\omega - \delta)$, leads to the same values of τ and X for the reflected wave packet.

The apparent distance 'travelled' by the maximum of the transmitted wave packet inside the barrier is $(X^2+Z^2)^{1/2}$. Light *in vacuo* would pass this distance in a time $(X^2+Z^2)^{1/2}/c$. Therefore, for $\tau \leqslant (X^2+Z^2)^{1/2}/c$ the propagation of the maximum of the transmitted wave packet across the barrier can be considered as superluminal, the effective group velocity

$$c_g(\theta) = (X^2+Z^2)^{1/2}/\tau \quad (2.8)$$

being greater than c. From the relation $c_g(\theta) = c$ the threshold thickness of the barrier, $Z_{\min}(\theta)$, can be determined at which the transmission becomes superluminal. In the limit of critical incidence, $\theta \to \theta_c = \sin^{-1}(n_1/n)$, for a vacuum gap in glass ($n_1 = 1$, $n = 1\cdot5$) we obtain $Z_{\min}(\theta_c) = 0\cdot22\lambda$ for TE polarization, λ being the vacuum wavelength. Evidently, very narrow gaps are already 'superluminal' at critical incidence. Numerical calculations show that for $\theta > \theta_c$ the threshold thickness $Z_{\min}(\theta) > Z_{\min}(\theta_c)$.

The maximum of the reflected wave packet does not propagate superluminally, the maximal value of the ratio X/τ, ie $(X/\tau)_{Z\to\infty} = c/n \sin\theta$, not exceeding c.

3. Signals

The method of stationary phase as employed in the preceding section is only a first step in general techniques based on Fourier decomposition of waveforms (Brillouin 1960, Fox *et al* 1970). The explicit analytic evaluation of transmitted wave-packet integrals (2.5) is seriously complicated by the presence of the transmission coefficient which is an involved function of variables ω and θ, and can be implemented only for most simple waveforms. On the other hand an absolute upper bound for the signal velocity can be defined by the use of waveforms with a sharp front. So we choose a signal of the form of a plane wave with a sharp front

$$\begin{aligned} A &= \exp(-i\omega t') \qquad && t' > 0 \\ A &= 0 && t' < 0 \end{aligned} \quad (3.1)$$

where a new variable has been introduced: $t' = t-(n/c)(x\sin\theta+z\cos\theta)$. The Fourier transform will be defined as

$$\mathscr{A}(p) = (2\pi)^{-1/2}\int_{-\infty}^{\infty} A(t')\exp(\mathrm{i}pt')\,\mathrm{d}t' = \mathrm{i}(2\pi)^{-1/2}(p-\omega)^{-1}. \tag{3.2}$$

The constituent plane waves differ in frequency but propagate all in the same direction. This exclusion of the variable θ saves a great deal of mathematical complexity but leads to some difficulties which will be discussed later.

The incident signal can be reconstructed as

$$A(t') = (2\pi)^{-1/2}\int_{-\infty}^{\infty}\mathscr{A}(p)\exp(-\mathrm{i}pt')\,\mathrm{d}p$$

and the transmitted signal (for $z > Z$) as

$$\begin{aligned}A_{\mathrm{T}}(t') &= (2\pi)^{-1/2}\int_{-\infty}^{\infty}\mathscr{A}(p)T(p,\theta)\exp(-\mathrm{i}pt')\,\mathrm{d}p\\ &= \mathrm{i}(2\pi)^{-1}\int_{-\infty}^{\infty}(p-\omega)^{-1}[\sin\delta/\sin(\delta-\mathrm{i}\alpha p)]\exp(-\mathrm{i}pt'-\mathrm{i}nZp\cos\theta/c)\,\mathrm{d}p.\end{aligned} \tag{3.3}$$

The integration is carried out by closing the contour of integration in the complex plane. The integrand has simple poles at $p_{\mathrm{I}} = \omega$ (originating from $\mathscr{A}(p)$) and $p_m = \mathrm{i}(m\pi-\delta)/\alpha$, $m = 0, \pm1, \pm2, \ldots$ (originating from $T(p,\theta)$). The contour is closed by an infinite semicircle in the upper half-plane for $t'' = t'-nZ\cos\theta/c < 0$ and in the lower half-plane for $t'' > 0$. In the first case poles at p_m with $m = 1, 2, \ldots$ are enclosed and in the second case poles at p_m with $m = 0, -1, -2, \ldots$ and the pole at p_{I}. Though the latter lies on the real axis, it can be shown to belong to the lower half-plane by the well-known device of an additional exponential factor $\exp(-\eta t')$ with a positive constant $\eta\to 0$ in the incident wave (3.1) to ensure its regularity for $t'\to\infty$.

The sums of pole residues give

$$\begin{aligned}A_{\mathrm{T}} &= -\sin\delta\sum_{1}^{\infty}(-1)^m(m\pi-\delta+\mathrm{i}\alpha\omega)^{-1}\exp[(m\pi-\delta)t''/\alpha] && t''<0\\ A_{\mathrm{T}} &= T(\omega,\theta)\exp(-\mathrm{i}\omega t')\\ &\quad+\sin\delta\sum_{-\infty}^{0}(-1)^m(m\pi-\delta+\mathrm{i}\alpha\omega)^{-1}\exp[(m\pi-\delta)t''/\alpha] && t''>0.\end{aligned} \tag{3.4a}$$

After a somewhat lengthy procedure the above expressions can be brought to a common form

$$A_{\mathrm{T}} = \alpha^{-1}\sin\delta\exp(-\mathrm{i}\omega t'')\int_{-\infty}^{t''}\exp(\mathrm{i}\omega s-s\delta/\alpha)[1+\exp(-\pi s/\alpha)]^{-1}\,\mathrm{d}s. \tag{3.4b}$$

The same form can more easily be obtained from equation (3.3) by use of the convolution theorem

$$A_{\mathrm{T}} = (2\pi)^{-1}A(t'')*\mathscr{T}(t'',\theta),$$

where $\mathscr{T}(t'',\theta)$ represents the inverse transform of the transmission coefficient.

4. Discussion

Due to the presence of poles of $\mathscr{A}_T(p)$ in the upper half-plane the transmitted waveform does not in general exhibit a sharp front and cannot, according to Fox *et al* (1970), be considered a proper signal. Indeed, in the limit of $t'' \to -\infty$ we obtain

$$A_T \simeq \sin\delta(\pi-\delta+i\alpha\omega)^{-1}\exp[(\pi-\delta)t''/\alpha],$$

ie a small non-oscillating field preceding the signal. This field is the consequence of the simple form of the incident signal and can qualitatively be explained as follows: the front of the incident signal penetrates the barrier at all times, the point of intersection travelling along the boundary plane at $z = 0$ with the velocity $c/n \sin\theta$. The disturbance inside the barrier travels along it with the local velocity c/n_1. For angles of incidence greater than θ_c, the disturbance from earlier times thus always precedes the advent of the wave front to the boundary plane at $z = Z$. Consequently it generates a field in the medium beyond the barrier before the proper transmitted signal appears, and smears out its front. However, at the critical angle itself both velocities are equal and a sharp front of the transmitted signal can nevertheless be expected. Indeed, Fox's criterion is fulfilled in this case as all the poles at p_m with $m \neq 0$ recede to infinity with vanishing residues. The remaining poles at p_I and p_0 give rise to the signal

$$(A_T)_{\theta=\theta_c} = \beta[\exp(-i\omega t'')-\exp(-\beta t'')]/(\beta-i\omega) \qquad t''>0$$

$$(A_T)_{\theta=\theta_c} = 0 \qquad t''<0$$

with $\beta = (\delta/\alpha)_{\theta=\theta_c} = 2nc/a^2Z\cos\theta_c$.

Evidently the signal is distorted. Let us examine the quantity A^*A, which is proportional to the energy flux in the signal and represents a suitable real envelope of the signal. The step function of the incident signal is distorted to (figure 1)

$$(A_T^*A_T)_{\theta=\theta_c} = \beta^2[1-2\cos\omega t''\exp(-\beta t'')+\exp(-2\beta t'')]/(\omega^2+\beta^2),$$

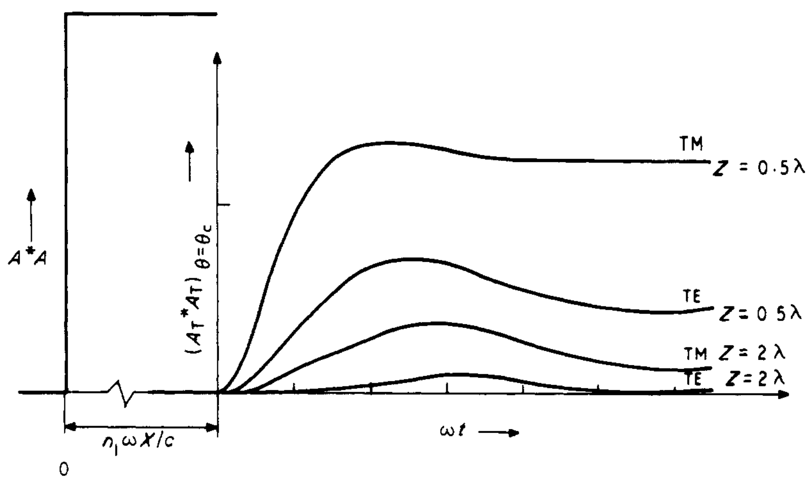

Figure 1. The time dependence of $(A_T^*A_T)_{\theta=\theta_c}$, proportional to the energy flux in the transmitted signal at critical incidence.

ie to a function which grows from zero at $t'' = 0$ proportionally to t''^2 and approaches asymptotically the expected value $T^*T = \beta^2/(\omega^2+\beta^2)$ at $t'' \to \infty$. This distortion is the consequence of the dispersion property of the transmission coefficient.

In spite of a sharp front the time of passage of the signal across the barrier cannot be determined by itself because of the translational invariance of the solution. That is, there is no indication as to which point of the boundary plane $z = Z$ represents the exit point of the ray, incident at $x = 0$, $z = 0$. However, borrowing the result for the shift X from § 2, an effective signal velocity of barrier penetration can be defined analogously to the effective group velocity (2.8) as

$$c_s = (X^2+Z^2)^{1/2}/|t''(X, Z)|. \tag{4.1}$$

For critical incidence $X \to \infty$ and $|t''(X, Z)| = n_1 X/c$ and consequently

$$c_s(\theta_c) = c/n_1 \leqslant c,$$

ie the signal velocity equals the phase velocity of light inside the barrier: the signal does not propagate superluminally.

There is little to be said at present about the propagation of signals for angles of incidence greater than the critical angle. The preceding non-oscillatory field could be avoided by use of an incident wave with a sharp side edge (eg Ricard 1973) as well as the sharp front employed here. The fact that the barrier at angles of incidence $\theta > \theta_c$ is less superluminal with respect to the effective group velocity than at critical incidence indicates that the present proof for θ_c should successfully be extended there.

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